Savings Facts & Formulae

Information Sheet

Useful Facts and Formulae

The amount invested is called the **principal**. It is often represented by the letter P, but can also be represented by other letters. The interest rate is usually represented by r or R and the time period by n (months or years).

When a principal £P earns compound interest at an annual rate R for n years, the **final amount** in the account is:

$$A = P(1+R)^n$$

The **annual rate** at which a principal P would increase to an amount A after n years is:

$$R = \sqrt[n]{\frac{A}{P}} - 1$$

Other time periods

You can use the above formulae for other time periods. For example, the monthly rate can be used instead of R with the number of months instead of n.

Annual Equivalent Rate (AER)

In practice, different accounts add interest at different rates and different time intervals. To make comparisons easier, all advertisements for savings accounts give the AER. This is what the interest rate would be if interest was paid and compounded once each year.

$$\text{AER} = \frac{\text{Interest earned in 1 year}}{\text{Amount in the account at the beginning of the year}} \times 100\%$$

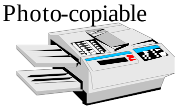
The AER that corresponds to a rate r added n times per year is given by $\text{AER} = (1+r)^n - 1$

Notes

The rates R and r should always be substituted as **decimals**.
Be as accurate as you can in your calculations. Do not round intermediate values - use your calculator's memory where necessary.

Example

The formula $A = P(1+R)^n$ gives the amount accruing when a principal £P earns compound interest at the annual rate R for n years. Neil invests £2000 at the fixed annual rate of 4.2%. Calculate the amount in Neil's account after 10 years.



Solution

To write the rate 4.2% as a decimal, divide by 100 to give $R = \dfrac{4.2}{100} = 0.042$

Substituting this with $P = 2000$ and $n = 10$ into the formula gives:

$$A = 2000(1+0.042)^{10} = 2000\times1.042^{10}$$
$$= 3017.916\ldots$$

Use the power key to work this out on your calculator

Amount in the account after 10 years = £3017.92 (to the nearest pence)

Example

Kate invests £S. Interest is paid at the fixed rate of 0.35% per month.
After n years, the amount of money which Kate will have as a result of this investment is £P, where P is given by $P = S\times1.0035^{12n}$

a) Kate invests £6000. Find the amount of money she will have at the end of 1 year.
b) Hence find the AER (Annual Equivalent Rate) for this investment.

Solution

a) Substituting $S = 6000$ and $n = 1$ into the formula gives:

$$P = 6000\times1.0035^{12}$$
$$= 6256.908\ldots$$

Use the power key to work this out on your calculator

Amount in the account at the end of the year = £6256.91 (to the nearest pence)

b) $$\text{AER} = \frac{\text{Interest earned in 1 year}}{\text{Amount in the account at the beginning of the year}}\times100\%$$

In this case the interest earned = £6256.91 – £6000 = £256.91

$$\text{AER} = \frac{256.91}{6000}\times100\% = 0.04281\ldots$$

The AER for this investment = 4.28% (to 3sf)

Example

The annual rate, R, expressed as a decimal, at which a principal £P would increase to an amount £A after n years is given by the formula $R = \sqrt[n]{\dfrac{A}{P}} - 1$

An investment of £3500 has grown to £4600 after four years.
Find the annual percentage rate of interest.

Solution

Substituting $P = 3500$, $A = 4600$ and $n = 4$ into the formula gives

$$R = \sqrt[4]{\frac{4600}{3500}} - 1 = \sqrt[4]{1.31428...} - 1 = 1.07071... - 1 = 0.07071\ldots$$

To write this as a %, multiply by 100

The annual percentage rate of interest = 7.07% (to 3sf)

Some to try

1 The formula $A = P(1+R)^n$ may be used to find the amount accruing when a principal £P earns compound interest at the annual rate R for n years.
Moira invests £5000 at the annual rate of 5.2% paid compound annually.
Calculate the amount in Moira's account after 3 years.

2 Seth invests £S. Interest is paid at the fixed rate of 0.29% per month.
After n years, the amount of money which Seth will have as a result of his investment is £P, where P is given by $P = S \times 1.0029^{12n}$
(a) Seth invests £10 000 for 1 year. Use the formula to find the total amount of money which Seth will have at the end of the year.
(b) Hence find the AER (Annual Equivalent Rate) for this investment.

3. The annual rate, R, expressed as a decimal, at which a principal £P would increase to an amount £A after n years is given by the formula $R = \sqrt[n]{\frac{A}{P}} - 1$
An investment of £3750 grows to £4725 in 5 years.
Find the annual rate of interest on this investment, expressed as a percentage.

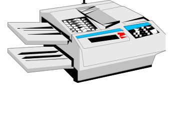
4. The interest, £I, given when you invest a sum of money, £S, for n years at a fixed rate of interest of 6% is given by $I = S(1.06^n - 1)$.
Find the interest earned by an investment of £750 invested in this way for 8 years.

5. Rowan invests £S at a fixed rate of interest. Interest is paid at 0.65% every two months.
After n years, the amount of money which Rowan will have as a result of his investment is £P, where P is given by $P = S \times 1.0065^{6n}$
(a) Rowan invests £20 000 for 1 year. Use the formula to find the total amount of money which Rowan will have at the end of the year.
(b) Hence find the AER (Annual Equivalent Rate) for this investment.

6. The annual rate, R, expressed as a decimal, at which a principal £P would increase to an amount £A after n years is given by the formula $R = \sqrt[n]{\frac{A}{P}} - 1$
Find the annual percentage rate of interest if £4500 grows to £5645 in three years.

7. The AER corresponding to a rate r added n times per year is given by $\text{AER} = (1+r)^n - 1$
Find the AER corresponding to 0.35% added each month.

8. Lily invests £S at a fixed rate of interest. The interest is paid quarterly at the rate of 0.6% per quarter. After n years, the amount of money which Lily will have as a result of her investment is £P, where P is given by $P = S \times 1.006^{4n}$
(a) Lily invests £3000 for 1 year. Use the formula to find the total amount of money which Lily will have at the end of the year.
(b) Hence find the AER (Annual Equivalent Rate) for this investment.



9. The annual rate, R, expressed as a decimal, at which a principal £P would increase to an amount £A after n years is given by the formula $R = \sqrt[n]{\frac{A}{P}} - 1$
An investment of £40 000 has grown to £49 254 after five years.
Find the annual percentage rate of interest on this investment.

10. The amount of money, £P, you will have in an account when you have invested a sum of money, £S, for n months at a fixed rate of interest of 0.3% per month is given by
$$P = S \times 1.003^n$$
(a) Find the total amount of money Neil will have in his account if he invests £2500 for 1 year at 0.3% interest per month.
(b) Hence find the AER (Annual Equivalent Rate) for this investment.

11. The interest, £I, given when you invest a sum of money, £S, for n years at a fixed rate of interest of 6.5% is given by $I = S\left(1.065^n - 1\right)$.
Find the interest Chloe will earn when she invests £20 000 in this way for a total of 6 years.

12. The monthly rate, r, expressed as a decimal, at which a principal £P would increase to an amount £A after n months is given by the formula $r = \sqrt[n]{\frac{A}{P}} - 1$
An investment of £2100 has grown to £2281 after a year.
Find the monthly rate of interest on this investment, expressed as a percentage.

13. The formula $A = P(1+r)^n$ may be used to find the amount accruing when a principal £P earns compound interest at the monthly rate r for n months.
William invests £5000 at the monthly rate of 0.52% compounded each month.
Calculate the amount in William's account after 2 years.

14. Amy invests £S. Interest is paid at the fixed rate of 0.18% per month.
After n months, the amount of money which Amy will have as a result of her investment is £P, where P is given by $P = S \times 1.0018^n$
(a) Amy invests £2000 for 1 year. Use the formula to find the total amount of money which Amy will have at the end of the year.
(b) Hence find the AER (Annual Equivalent Rate) for this investment.

15. The formula $R = (1+r)^n - 1$ gives the AER corresponding to a rate r added n times per year.
a) Find the AER corresponding to 0.5% added each month.
b) Find the AER corresponding to 1.5% added each quarter.
c) Find the AER corresponding to 3% added every 6 months.

16. Kerry invests £S at a fixed rate of interest. Interest is paid at 1.2% every four months.
After n years, the amount of money which Kerry will have is £P, where P is given by
$P = S \times 1.012^{3n}$
(a) Kerry invests £2500 for 1 year. Use the formula to find the total amount of money which Kerry will have at the end of the year.
(b) Hence find the AER (Annual Equivalent Rate) for this investment.

Teacher Notes

Unit Intermediate Level, ***Calculating Finances***

Skills used in this activity:
- substituting values into formulae associated with savings
- using a calculator to evaluate expressions.

Preparation
Students need to be able to use a calculator to evaluate expressions involving brackets, powers and roots. The accompanying Powerpoint presentation could be used for class discussion about formulae and to go through examples of this type before students try some themselves. This presentation can be adapted to include more or fewer examples.

Answers (to 3 sf)

1.	£5821.26		
2.	(a) £10 353.60	(b) 3.54%	
3.	4.73%		
4.	£445.39		
5.	(a) £20 792.79	(b) 3.96%	
6.	7.85%		
7.	4.28%		
8.	(a) £3072.65	(b) 2.42%	
9.	4.25%		
10.	(a) £2591.50	(b) 3.66%	
11.	£9182.85		
12.	0.691%		
13.	£5662.78		
14.	(a) £2043.63	(b) 2.18%	
15.	(a) 6.17%	(b) 6.14%	(c) 6.09%
16.	(a) £2591.08	(b) 3.64%